DOCUMENT RESUME

ED 171 548 SE 027 684

AUTHOR Sachar, Jane; And Others
TITLE Error Analysis on Literals and Numerals in Solving Equations.
PUB DATE 79
NOTE 10p.; Paper presented at the annual meeting of the American Educational Research Association (San Francisco, California, April 8-12, 1979)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Algebra; *Error Patterns; *Instruction; *Mathematics Education; Postsecondary Education; *Research; *Teaching Techniques
IDENTIFIERS *Equations (Mathematics)

ABSTRACT

This study compared errors made while solving equations using literals (letters) with errors made while solving the same equations using numerals. The subjects were 54 Navy students enrolled in a remedial mathematics course. Error data were analyzed by means of coding all errors into ten major categories, each containing subcategories. Students made significantly more errors in solving literal equations than in solving numeral equations. (MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Meryl S. Baker
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

ED171548

Error Analysis on Literals and Numerals in Solving Equations

Jane Sachar, Meryl S. Baker, Bobby G. Miller

Navy Personnel Research and Development Center

Algebraic equations may be represented in the general form using literals (alphabetic letters) as both coefficients and constants. It is generally accepted that students have more difficulty solving equations containing literals than solving equations containing numerals. Unfortunately, research documenting this fact is rarely, if ever, reported. It has been suggested, however, that the increased difficulty arises due to (1) less familiarity with literals than with numerals, (2) inability to comprehend the abstract nature of literals, (3) inability to fully comprehend that literals, like numerals, are mere representations which take the place of values, and (4) inability to differentiate between the known constants and the value to be solved for when all variables are literals. (For example, when solving for x in the equation, $2x + 3 = 9$, the goal is clear; x is the only literal and must appear alone on one side of the equation. However, in the equation $ax + b = c$, all symbols excluding the operation and the notation for equality are literals, and it may be unclear to the student how x differs from a, b, and c.)

The purpose of the present study was to compare errors made while solving equations using literals with errors made while solving the same equations using numerals.

SE 027 084

Method

The subjects participating in this study were 54 Navy students enrolled in a remedial mathematics course. Almost all students had taken a high school algebra course prior to entering the Navy.

Each student was given the following two sets of nine problems:

NUMERALS		LITERALS	
1)	$f - 4 = 12$	1)	$f - a = b$
2)	$4 \cdot f = 23$	2)	$c \cdot f = d$
3)	$9/f = 12$	3)	$\frac{k}{f} = m$
4)	$\frac{f - 4}{5} = 3$	4)	$\frac{f - n}{p} = r$
5)	$3 \cdot f + 8 = 21$	5)	$n \cdot f + g = h$
6)	$\frac{f - 2}{4} + 11 = 17$	6)	$\frac{f - b}{d} + g = a$
7)	$\frac{5}{9 - f} = 1$	7)	$\frac{c}{g - f} = 1$
8)	$\frac{12}{f} - 4 = 4$	8)	$\frac{n}{f} - m = p$
9)	$3(15 + 4 \cdot f) = 9$	9)	$a(h + g \cdot f) = c$

The two sets, each of which appeared on a separate page, required identical operations, but could be differentially characterized by the symbols used. Both sets of problems required solution of a literal variable. One set contained numerals as the remaining variables; the other contained literals.

The nine parallel sets of equations were formulated based on solution complexity. Two parallel sets of equations required one operation.(e.g., f - 4 - 12); three parallel sets of equations required two operations (e.g., $3 \cdot f + 8 = 21$); and four parallel sets of equations required three operations (e.g., $\frac{n}{f} - m = p$). The items were sequenced according to number of operations required. One step problems occurred first, two step second, and three step third.

Tests were administered by student proctors. Students were requested not to use "trial and error" strategies and were informed that their scores would not be recorded as the test was being administered and scored for research purposes only.

Results

Scores for solution of equations with literals ranged from 0 to 9 with a mean of 4.22 and a standard deviation of 2.625. Scores for solution of equations with numerals ranged from 2 to 9 with a mean of 5.72 and a standard deviation 2.491.

Error data were analyzed by means of coding all errors into the following major and sub categories:

CATEGORIES

00 - Slight variations from correctness

- 01 - return to correct configuration after error
- 02 - continue correctly from altered problem statement
- 03 - start over correctly
- 04 - reduction error
- 05 - careless recording
- 06 - substitutes values before/without isolating
- 07 - checks answer by substitution
- 08 - confirms wrong answer by substitution in altered equation

10 - Multiplication by zero not equal zero

20 - Order of operations

- 21 - add/subtract before multiply/divide
- 22 - introduce parentheses improperly
- 23 - restatement of problem changes order
- 24 - violates parentheses

30 - Division by zero

40 - Interchange multiply/divide or add/subtract

- 41 - multiply on one side, divide on other
- 42 - divide denominator

50 - Illegal cancellation

- 51 - clear numerator by multiplication
- 52 - clear numerator by division
- 53 - divide on one side only
- 54 - cancels in quotient

60 - Adding/subtracting fractions

- 61 - add/subtract fraction and whole
- 62 - subtract from denominator

CATEGORIES - continued

70 - Partial distribution

- 71 - partial distribution by multiplication (no parentheses)
- 72 - partial distribution by division (no parentheses)
- 73 - 'over' distributes (multiply/divide both terms of product)
- 74 - partial distribution with parentheses in place

80 - Confusion of operators

- 81 - confuse multiplication and division
- 82 - confuse negative sign and negative exponent
- 83 - confuse division and subtraction
- 84 - confuse multiplication and addition
- 85 - misuse of divide symbol ÷
- 86 - treat difference as product
- 87 - inconsistent use of operator

90 - Miscelleneous errors

- 91 - arithmetic error
- 92 - sign error
- 93 - solve for wrong variable
- 94 - sloppy work - can't follow
- 95 - invert fraction
- 96 - miscopy problem

Students made significantly more errors in solving literal equations than in solving numeral equations ($t = 7.40$, $p<.001$).

As complexity of problems increased, students made significantly more errors; and at all levels made more errors when solving the literal equations than when solving the numeral equations (least complex $t = 1.97$, $p<.05$; more complex $t = 2.83$, $p<.005$; ,ost complex $t = 7.14$, $p<.001$).

At all complexity levels, students used a greater number of steps in solving numeral equations than literal equations ($t = 6.76$, $p<.001$). However, this may be due to the fact that students were less likely to complete the literal equations. Also, solving numeral equations could involve the additional step of reduction, not possible with literals..

There were only three error classes where a significant difference was found between the number of errors made by students in solving literal equations and the number of errors made by students in solving numeral equations. They were:

Substitution ($t = -2.14$, $p<.03$)

Partial Distribution ($t = -3.35$, $p<.001$)

Confusing Operators ($t = -2.27$, $p<.02$)

In all three cases students solving the literal equations made these errors more often.

Frequency of error was analyzed in an attempt to isolate potential solution strategies. No overall error pattern could be discerned. Errors were unpredictable, which made error analysis through diagramming virtually impossible. An attempt was made, however, to discover a solution pattern by examining which variable was most often transposed first in solving for literals versus numerals. A three step problem was examined and no significant differences were found. Unfortunately, sufficient time was unavailable for further analysis of this type.

Discussion

Why did students make more errors when solving literal equations than when solving numeral equations? A number of reasons come to mind. First, it is easier for students to "check" their answers when solving numeral equations. Illogical errors such as:

$$1/2 + 1/2 = 1/4$$

are more likely to be caught and changed.

Further, numerical equations are usually simplified as one progresses through the steps. The equation solution below on the left decreases in succeeding steps from 4 terms to 3, and from 3 terms to 2.

The equation solution on the right remains constant, in relation to number of terms, throughout all steps.

Numeral Equation	Literal Equation
$3x + 6 = 12$	$ax + b = c$
$3x = 6$	$ax = c - b$
$x = 2$	$x = \frac{c - b}{a}$

This simplification process could make the students encounter with each succeeding step less difficult as a consequence of the number of terms which need to be processed and manipulated. If, however, the literal equation contained only multiples of one literal term, then each succeeding step would be simplified as a consequence of factoring. This may result in equalizing the complexity of the literal problem with that of the numeral problem.

Finally, students are probably far more comfortable with the concrete nature of numerals than the abstract nature of literals. It is easier for most students to conceptualize "12 sticks minus 6 sticks equal 6 sticks" than for them to conceptualize "a sticks minus b sticks equal a minus b sticks." The fact that the types of errors which occurred with significantly greater frequency were related to the abstract nature of the literal problems (substitution, partial distribution, and confusion of operators) lends credence to this argument.

The data coding for this large a group took an enormous amount of time and effort and no consistent error patterns were uncovered. The error analysis procedure would appear to be of most value in situations where errors made by individual students could be isolated and remediation recommended. In addition, continued research in the area of correct solution paths may also be profitable.

Finally, data from this study which indicate that students possess greater facility in solving numeral equations than they do in solving literal equations may have implications for teaching rules for equation solving. An ability to learn and use the rule in its general form, with literals, would decrease the length of time required for instruction, by reducing the number of numerical examples used in presentations and student practice. Thus, it may be worthwhile to explore various means of improving student understanding and facilitation with literal terms.